

Measuring the Average Period of Production

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Introduction

- **Production takes time:** from extraction and preparation of raw materials, to the design, manufacturing, assembly, and distribution of finished goods
- Importance of production time in production emphasized by Classical economists and at core of **Böhm-Bawerk's (1889)** capital theory
 - ▶ Central concept: **average period of production** (APP)
 - ▶ APP = weighted average temporal distance between the time at which a firm employs its inputs and the time at which these inputs deliver finished goods to consumers
- Under stationarity conditions, we show that the APP corresponds to the ratio of a firm's inventory stock to the cost of the goods it sells (COGS)
- We compute APP using data from publicly traded companies worldwide
- Consistent with theory, APP increases as the cost of capital faced by firms decreases

Literature Review

- 'Austrian' capital theory
 - ▶ Jevons (1871), Böhm-Bawerk (1889), Wicksell (1934), Hicks (1939), Metzler (1950), Dorfman (1959), Findlay (1978), Kim and Shin (2017), Antràs (2023)
- Measuring the Average Period of Production
 - ▶ Theory: Böhm-Bawerk (1889), Hicks (1934), Malinvaud (2003)
 - ▶ Empirics: Schwartzman (2014)
- Inventories, Just-in-Time: Alessandria et al. (2011), Schwartzman (2014), Ferrari (2022), Pisch (2022), Carreras-Valle (2022)

Conceptual Framework

Time and Production

- Consider a production process taking place in continuous time over a potentially endogenous time interval $[0, T]$
- Mapping between inputs and output is governed by the production function

$$Y(\mathbf{L}) = Z(\mathbf{L}) F(\mathbf{L}),$$

- $\mathbf{L}$ is the infinitely-dimensional vector of inputs employed along the production process
- Firms treat input cost w and good price p as given (easy to let wages be time-varying)
- Function $F(\mathbf{L})$ captures a production technology: assumed homogeneous of degree 1
- Function $Z(\mathbf{L})$ captures productivity, which may also potentially be affected by path $F(\mathbf{L})$ (but assume homog. of degree 0 to avoid scale effects)

The Average Period of Production

- Define the vector $\lambda = \{\ell(t)/L\}_{t \in [0, T]}$, where $L = \int_0^T \ell(t) dt$
- $\lambda(t)$ captures the distribution of input expenditures along the production process
- We define the 'average period of production' or $\mathcal{APP}$ as

$$\mathcal{APP}(\lambda) = \int_0^T (T - t) \lambda(t) dt.$$

- $\mathcal{APP}$ represents a weighted average temporal distance between the time $t \in [0, T]$ at which inputs are utilized and the time T marking the completion of the process and the sale of the good
- $\mathcal{APP}$ is higher when inputs are used earlier in production and (typically) when T is larger
- Easy to build an expenditure-based $\mathcal{APP}$ if input costs vary along the chain (Hicks, 1939)

The Benefits of 'Longer' Production Processes

- Böhm-Bawerk (1889) posited that more 'roundabout' production processes tend to be associated with higher input/labor efficiency
- Roundabout production features a disproportionately large share of input expenditures happening far from the completion and sale of the product, i.e., a higher APP
- Assume that the function $Z(L)$ depends on L in a way that makes Z increasing in the average period of production, or $Z(L) = Z(APP)$, with $Z'(APP) > 0$.
- **Examples:** timber, wine, need for care and precision in complex manufacturing, R&D, clinical trials, etc.

Determining the Average Period of Production

- Firm chooses the input vector $\mathbf{L}$ and the length of the interval $[0, T]$ to maximize

$$\max_{\mathbf{L}, T} \pi = pZ(\mathbf{L})F(\mathbf{L})e^{-rT} - w \int_0^T \ell(t) e^{-rt} dt,$$

- With differentiable functions $Z(\mathbf{L})$ and $F(\mathbf{L})$, and imposing zero profits, we have:

$$\frac{\partial Z(\mathbf{L})F(\mathbf{L})}{\partial \ell(t)} = \frac{w}{p} e^{r(T-t)}.$$

- Given two inputs at two different points in time, say $t_H > t_L$, we have that

$$\frac{\ell(t_H)}{\ell(t_L)} = \frac{\varepsilon_{Y, \ell(t_H)}}{\varepsilon_{Y, \ell(t_L)}} e^{r(t_H - t_L)},$$

- Larger input use in more important 'stages', but desire to backload input expenditures, and more so the higher the interest rate r (negative impact of r on $\mathcal{APP}$)

Determining the Average Period of Production (II)

- Turning to the first-order condition with respect to T , we have

$$p \frac{\partial (Z(\mathbf{L}) F(\mathbf{L}))}{\partial T} = w\ell(T) + rpZ(\mathbf{L}) F(\mathbf{L})$$

which can be succinctly expressed as:

$$T = \frac{\varepsilon_{Y,T}}{\alpha_T + r}, \quad (1)$$

where $\varepsilon_{Y,T}$ is the elasticity of output with respect to T (holding the vector $\mathbf{L}$ fixed) and where $\alpha_T = w\ell(T) / (pZ(\mathbf{L}) F(\mathbf{L}))$

- This equation hints at a negative relationship between T and r , which in turn suggests an **additional channel by which higher cost of capital reduces APP**
- **In paper:** three examples that formally demonstrate a negative impact of r on APP

A Measure of The Average Period of Production

Toward a Measure of the Average Period of Production

- Cumulated cost of a good in an initial interval $[0, \hat{t}]$ of production is given by

$$C(\hat{t}) = \int_0^{\hat{t}} w\ell(t) dt$$

- Firms simultaneously carry out many production processes at various phases of completion
- At each instant t , the firm:
 - ▶ begins the production of N goods
 - ▶ continues to add value to goods began in previous periods $t' \in (t - T, t)$
 - ▶ completes the production of N goods began at $t - T$
- In steady-state, total inventories are

$$I = N \int_0^T C(\hat{t}) d\hat{t} = Nw \int_0^T \int_0^{\hat{t}} \ell(s) ds d\hat{t}$$

Toward a Measure of the Average Period of Production

- Ratio of total inventories to the cost of goods sold (COGS) is

$$\frac{I}{C(T)} = \int_0^T \int_0^{\hat{t}} \frac{\ell(s)}{L} ds d\hat{t} = \int_0^T \int_0^{\hat{t}} \lambda(s) ds d\hat{t}$$

- Solving this double integral by parts delivers

$$\frac{I}{C(T)} = \int_0^T \int_0^{\hat{t}} \lambda(s) ds d\hat{t} = \int_0^T (T - \hat{t}) \lambda(\hat{t}) d\hat{t} = \mathcal{APP}$$

Proposition

The average period of production $\mathcal{APP}$ associated with a firm's production process can be computed as the ratio of the firm's inventories to its cost of goods sold (COGS).

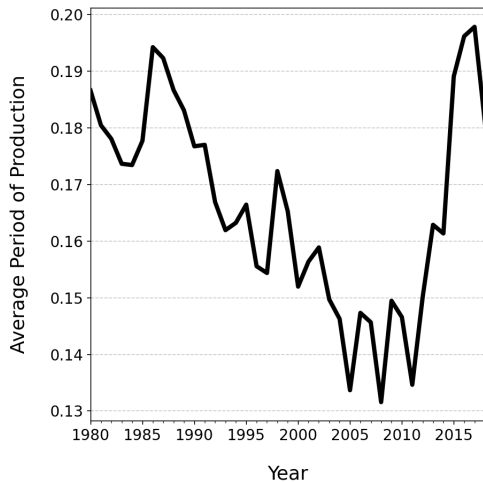
Data Sources

Data Sources

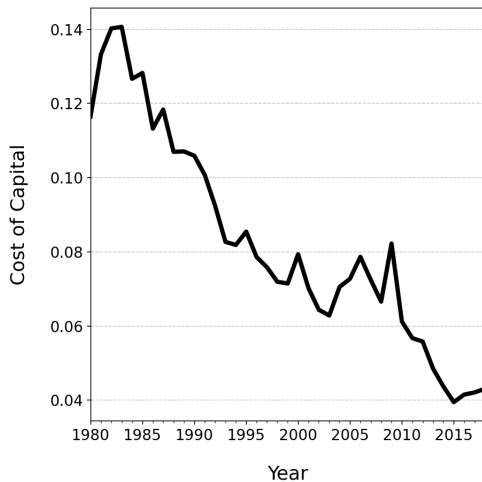
- Two key inputs — firm inventories and COGS — readily available for publicly traded companies
- We construct APP using annual financial reports from publicly traded U.S. and global firms for the period 1980–2018 ([Compustat](#))
- Estimate the cost of capital as ratio of interest expenses to the sum of long-term and short-term debt ([Compustat](#))
- Firm's industry classification standardized to 2012 NAICS codes (+ easily said than done)
- Add IT capital intensity ([BLS](#)) and other industry characteristics ([NBER-CES database](#))
- Largely focus on firms in Compustat classified as belonging to the manufacturing sector (NAICS code 31–32–33)

The Average Period of Production in US Manufacturing

Aggregate Trends in US Manufacturing



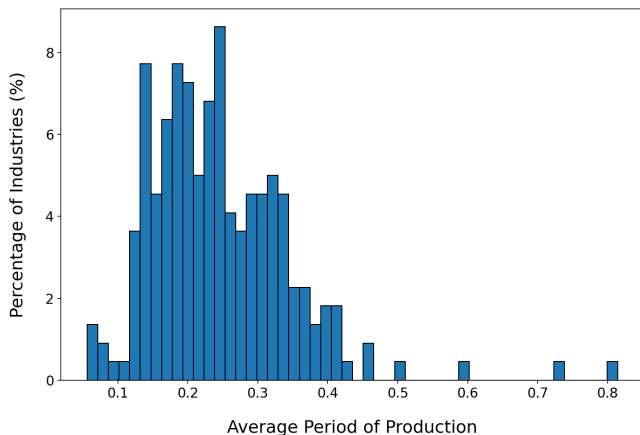
Cost-Weighted Average Production Period



Cost-Weighted Average Cost of Capital

Distribution of the Average Production Period in U.S. Manufacturing

- We compute APP for 220 six-digit HS sectors (cost-weighted averages after winsorizing)
 - ▶ Average APP is 0.311 years or 114 days (this is not T !)



Top 10 U.S. Industries based on the Average Period of Production

NAICS	Industry	Average Period of Production
312140	Distilleries	0.814
312130	Wineries	0.730
339115	Ophthalmic Goods Manuf.	0.594
312230	Tobacco Manuf.	0.511
333517	Machine Tool Manuf.	0.464
333132	Oil and Gas Field Machinery and Equipment Manuf.	0.458
316998	All Other Leather Good and Allied Product Manuf.	0.432
325412	Pharmaceutical Preparation Manuf.	0.418
332215	Metal Kitchen Cookware, Utensil, Cutlery, and Flatware Manuf.	0.415
334510	Electromedical and Electrotherapeutic Apparatus Manuf.	0.414

Bottom 10 U.S. Industries based on the Average Period of Production

NAICS	Industry	Average Period of Production
311511	Fluid Milk Manuf.	0.056
324110	Petroleum Refineries	0.068
311812	Commercial Bakeries	0.071
325193	Ethyl Alcohol Manuf.	0.073
336370	Motor Vehicle Metal Stamping	0.081
312112	Bottled Water Manuf.	0.096
327992	Ground or Treated Mineral and Earth Manuf.	0.112
311520	Ice Cream and Frozen Dessert Manuf.	0.118
327420	Gypsum Product Manuf.	0.119
327320	Ready-Mix Concrete Manuf.	0.119

Average Production Period and Other Industry Characteristics

Industry characteristic	Correlation with APP	p-Value
<i>Capital Intensity</i>		
Log (Capital Stock / Total Workers)	-0.1855	0.0058
Log (Capital Expenditures / Payroll)	-0.1928	0.0041
<i>Skill Intensity</i>		
Log (Non-Production Workers / Total Workers)	0.2838	0.0000
Non-Production Payroll / Payroll	0.3589	0.0000
<i>Inventory Intensity</i>		
Inventories / Sales	0.7060	0.0000
Inventories / (Payroll + Material Costs + Energy Costs)	0.8062	0.0000
Inventories / (Prod. Worker Wages + Material Costs + Energy Costs)	0.8032	0.0000
<i>Productivity</i>		
Log (Value Added / Total Workers)	0.0413	0.5427
Total Factor Productivity	0.0269	0.6911

The Average Period of Production and the Cost of Capital For US Firms

- Conceptual framework predicts that $\mathcal{APP}$ is negatively associated with the cost of capital
- Using panel data on publicly traded firms in U.S. manufacturing during the period 1980–2018, we estimate

$$\mathcal{APP}_{it} = \beta R_{it} + \gamma \mathbf{Z}_{it} + \mu_i + \lambda_t + \varepsilon_{it}, \quad (2)$$

- ▶ i and t denote firms and years, respectively
 - ▶ R_{it} is the cost of capital faced by firm i in year t
 - ▶ $\mathbf{Z}_{it}$ is a vector of industry-level controls for the sector to which firm i belongs
 - ▶ μ_i and λ_t are firm and year fixed effects, respectively
- All variables in logs so we estimate elasticities

The Average Period of Production and the Cost of Capital For US Firms

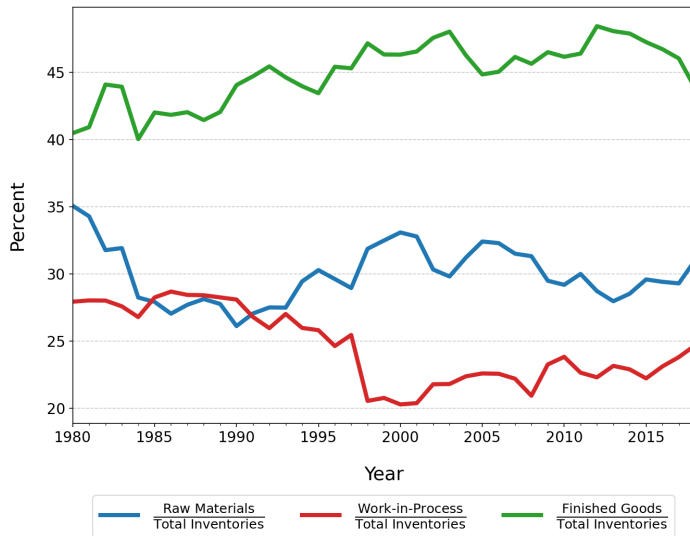
Dependent Variable:	Average Period of Production			
	(1)	(2)	(3)	(4)
Cost of Capital	-0.090 ^{***} (0.006)	-0.087 ^{***} (0.006)	-0.088 ^{***} (0.006)	-0.088 ^{***} (0.006)
IT Capital Intensity	—	0.018 (0.013)	0.022 (0.015)	0.022 (0.014)
Capital Intensity	—	—	-0.030 [*] (0.016)	-0.030 (0.021)
Skill Intensity	—	—	0.037 (0.039)	0.037 (0.035)
Labor Productivity	—	—	-0.038 (0.029)	-0.038 (0.033)
R^2	0.760	0.781	0.780	0.780
Observations	56,133	43,007	39,076	39,076
Clustering	Firm,Year	Firm,Year	Firm,Year	NAICS 4,Year

Back-of-the-Envelope Quantitative Interpretation

- How large is the coefficient -0.090 in economic terms?
- Between 2005 and 2018, the $\mathcal{APP}$ increased from 0.134 to 0.181, corresponding to a log change of 0.301
- Over the same period, the cost of capital faced by firms declined from 0.073 to 0.043, a log change of -0.529
- Based on our estimates, lower interest rates account for approximately 16% of the observed 0.301 log change in $\mathcal{APP}$ during this period
- But, admittedly, we are **not** estimating a causal relationship

The Average Period of Production by Inventory Type

Cost-Weighted Average Shares of Total Inventories in U.S. Manufacturing



Average Period of Production and Cost of Capital by Inventory Type

	Total Inventories to COGS Ratio (1)	Raw Materials Inventories to COGS Ratio (2)	Work-in-Process Inventories to COGS Ratio (3)	Finished Goods Inventories to COGS Ratio (4)
Cost of Capital	-0.101*** (0.006)	-0.100*** (0.008)	-0.128*** (0.012)	-0.108*** (0.009)
R^2	0.766	0.745	0.768	0.737
Observations	34,865	34,865	34,865	34,865

- Remarkably similar elasticities for all three types of inventories

The Average Period of Production in Global Manufacturing

Rank Correlations of Average Period of Production across Countries

Code	CAN	CHN	DEU	GBR	IND	JPN	KOR	MYS	TWN	USA
CAN	—	—	—	0.53 ^c	0.73 ^a	0.75 ^a	—	—	—	0.71 ^a
CHN	—	—	—	—	0.10	0.53 ^c	0.68 ^b	—	0.38	0.65 ^b
DEU	—	—	—	—	—	—	—	—	—	0.75 ^b
GBR	0.53 ^c	—	—	—	0.79 ^a	0.63 ^a	0.63 ^b	—	0.65 ^b	0.74 ^a
IND	0.73 ^a	0.10	—	0.79 ^a	—	0.47 ^a	0.55 ^a	0.37	0.20	0.41 ^a
JPN	0.75 ^a	0.53 ^c	—	0.63 ^a	0.47 ^a	—	0.75 ^a	0.15	0.38 ^b	0.58 ^a
KOR	—	0.68 ^b	—	0.63 ^b	0.55 ^a	0.75 ^a	—	—	0.52 ^b	0.67 ^a
MYS	—	—	—	—	0.37	0.15	—	—	—	-0.03
TWN	—	0.38	—	0.65 ^b	0.20	0.38 ^b	0.52 ^b	—	—	0.53 ^a
USA	0.71 ^a	0.65 ^b	0.75 ^b	0.74 ^a	0.41 ^a	0.58 ^a	0.67 ^a	-0.03	0.53 ^a	—

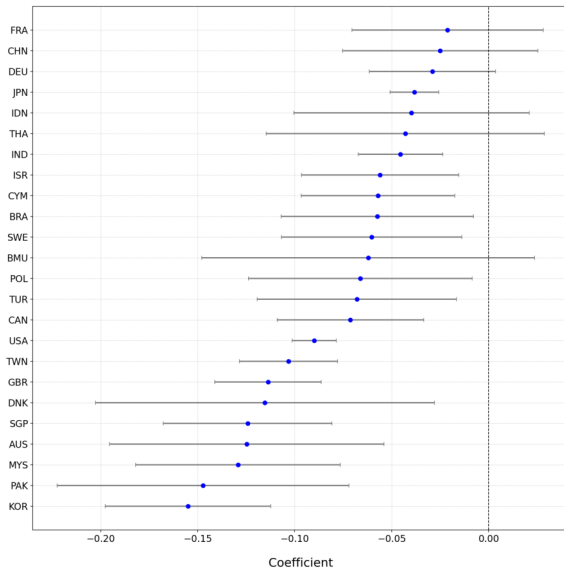
Notes: Spearman rank correlations are reported. Statistical significance at the 1%, 5% and 10% levels are denoted by subscripts *a*, *b* and *c*, respectively. Source: Compustat.

- Fairly stable rankings across countries (technological determinants of *APP* important)

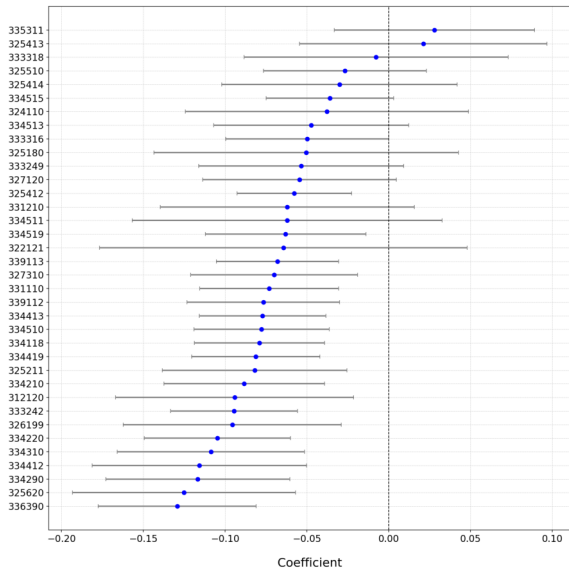
Average Period of Production and Cost of Capital For Global Firms

Dependent Variable:	Average Period of Production			
	(1) USA only	(2) USA only	(3) All Countries	(4) All Countries
Cost of Capital	-0.090*** (0.006)	-0.090*** (0.006)	-0.078*** (0.005)	-0.077*** (0.005)
R^2	0.760	0.760	0.765	0.770
Observations	56,133	55,207	209,606	189,914
Clustering	Firm, Year	NAICS 4, Year	Firm, Year	NAICS 4, Year

Average Production Period and Cost of Capital: Country Heterogeneity



Average Production Period and Cost of Capital: Industry Heterogeneity



Conclusions

Conclusions

- We develop a measure to capture the temporal dimension of production, building on the Austrian capital theory of Böhm-Bawerk
- We define the average period of production as a weighted average distance between time at which firm uses its inputs and time at which inputs deliver finished goods to consumers
- Under stationarity conditions, this measure corresponds to the ratio of a firm's stock of inventories to the cost of goods it sells
- Readily computable using data from publicly traded companies worldwide
- Consistent with theoretical predictions, firms facing higher capital costs tend to exhibit shorter average production periods
- Ultimate aim is to foster further empirical research into how production length and interest rates shape industrial structure

Appendix Slides

Summary Statistics for Firms in U.S. Manufacturing Industry, 1980-2018

	Mean	Std. Dev.	P25	Median	P75	Count
Average Period of Production	0.311	0.245	0.161	0.254	0.389	56,850
Cost of Capital	0.109	0.085	0.065	0.091	0.123	56,850
IT Capital Intensity	0.048	0.050	0.014	0.023	0.074	43,875
Capital Intensity	167.0	264.3	48.20	83.77	163.3	51,069
Skill Intensity	0.367	0.141	0.244	0.362	0.478	52,408
Labor Productivity	151.4	174.9	61.35	97.71	178.3	52,408

Notes: The table reports summary statistics for firm-year observations of U.S. publicly traded companies in the manufacturing industry (2012 NAICS codes 31–32–33) for the period 1980–2018. Capital intensity and labor productivity are expressed in \$1,000 per worker. See the Appendix for the data preparation algorithm. Sources: Bureau of Labor Statistics, Compustat, NBER-CES Manufacturing Database.